Applications of Small-World Networks to some Socio-economic Systems

A. S. Elgazzar
Mathematics Department, Faculty of Education
45111 El-Arish, Egypt
E-mail: elgazzar@mans.edu.eg

Abstract

Small-world networks (SWN) are found to be closer to the real social systems than both regular and random lattices. Then, a model for the evolution of economic systems is generalized to SWN. The Sznajd model for the two-state opinion formation problem is applied to SWN. Then a simple definition of leaders is included. These models explain some socio-economic aspects.

Keywords: Small-world networks; Evolutionary economic models; Opinion formation models.

1 Introduction

The diffusion of a new concept (a technology, an opinion, an information, a disease, ...) through social and economic systems is a complex process that typically forms waves or avalanches. Also, it usually displays rich dynamics that is attracting the interest of many mathematicians and theoretical physicists [1].

A social network has two main properties: clustering and small-world effect. Clustering means every one has a group of collaborators, some of them will often be a collaborator by another person. Small-world effect means the average shortest person to person (vertex to vertex) distance is very short compared with the whole size of the system (number of vertices).

Regular lattices display the clustering property only. On the other hand, random lattices display the small-world effect without clustering [2]. The concept of SWN introduced in [2,3] has shown to combine both features. A SWN is a connected ring with some shortcuts joining between some randomly chosen vertices are added with small probability ϕ. Also, this structure combines between both local and nonlocal interactions. This combination is observed in many real systems. Then it is a useful concept in modeling the real systems. We used this concept in modeling different systems [4-8]. Here our interest is restricted to apply the concept of SWN to a model for the evolution in economic systems [8] and to a sociophysics model [7].

2 A model for the evolution of economic systems in SWN

A real economic system is a population of agents each of them has a technological level (a_i). Every agent is assumed to interact with a group of collaborators (neighbours) obtaining payoffs (profits). The base payoff is assumed to be higher for the higher technological levels, and the payoffs have to be bounded. Also, if an agent is too advance or too backwards relative to his/her neighbours, he/she always has incompatibility costs.

Every agent has to update his/her technological level to obtain the best payoff. Then the population reaches a state at which each agent is satisfied with his/her payoff. Then the technological level of a randomly chosen agent is raised by a quantity $\Delta \in (0, 1)$. Usually the new technology has a cost. The neighbours of this agent updates their technological levels according to the new conditions. They have the possibility to accept or refuse the new technology. The new technology may diffuse through the whole population making a uniform front for cheap updating cost. On the other hand, if the cost is very high, only few agents can modify their levels. For intermediate cost values, the updating process continues through the population making an avalanche until another stable state is reached, and so on. The avalanche's size, s is the number of agents that updated their levels in a step. It has been shown that [9] the avalanches size and several aspects of social and economic systems can be described in terms of power law distribution as,

$$P(s) \propto s^{-\gamma}, \quad \gamma \geq 1. \tag{1}$$

There are two updating optimalities: Nash and Pareto defined as follows:

Definition 1. (Nash optimality): An agent selects the technological level that maximize his own payoff without regards to his/her collaborators.

Definition 2. (Pareto optimality): An agent selects the technological level that maximize the average payoff of his/her group (he/she and the collaborators).

Towards a theoretical approach to this phenomena, Arenas et al. [10] introduced the following payoff function:

$$\pi(a_i, a_j) = \begin{cases} a_i - k_1(1 - \exp -(a_i - a_j)) & \text{if } a_i \geq a_j, \\ a_i - k_2(1 - \exp -(a_j - a_i)) & \text{if } a_i < a_j, \end{cases} \qquad j = i \pm 1, \tag{2}$$

where k_1 and k_2 represent the incompatibility costs resulting from being too advance or too backwards, respectively. The payoff of the agent i, π_i is $\pi_i = \pi(a_i, a_{i-1}) + \pi(a_i, a_{i+1})$. Depending on this payoff function, Arenas et al. [10] construct a 1-dimensional (1-d) model using Nash optimality. But Pareto optimality usually implies less erratic behaviour than Nash optimality [5]. Also, it has been proven that the payoff from Pareto updating rule is generally higher or equal to that from Nash updating rule [11].

Then we have reconstructed the 1-d model of Arenas et al. using Pareto optimality [8]. It is shown that [8], the dynamics this model depends only on the quantity $k = k_1 - k_2$, which behaves as the updating cost. For $k \leq 2$, the avalanches are of the size of the whole population corresponding to the case of uniform front. In the limit $k \to \infty$, the agents behave independently, so avalanches become of size one. For intermediate values of the parameter k, the system is shown to be critical, see Fig. (1), where $k = 4$. The three regimes are clearly observed in real systems.

The total payoff of each agent is calculated and compared in two cases, Pareto optimality and Nash optimality. Applying the same conditions in the two cases, it is found that the total payoff from the Pareto optimality is significantly higher than that from the Nash optimality for all agents. This important result means that if every one in a population tries to maximize the payoff (profit, fitness, ...) of his/her group, the personal payoff of each of them may be more maximized than that of the case when every one tries to increase his/her own payoff individually.

Then the model is generalized to SWN with $\phi = 0.05$. The shortcuts are fixed beforehand. The updating rule is Pareto. The vertices that do not have

shortcuts behave similar to the 1-d model. If a vertex i has a shortcutting neighbour $\mathrm{sc}(i)$, then its payoff is calculated as follows:

$$\pi_i = \pi(a_i, a_{i-1}) + \pi(a_i, a_{i+1}) + \pi(a_i, a_{\mathrm{sc}(i)}). \tag{3}$$

In this case the average payoff of its group becomes:

$$\pi_{\mathrm{av}} = (\pi_{i-1} + \pi_i + \pi_{i+1} + \pi_{\mathrm{sc}(i)})/4. \tag{4}$$

The application of the model to SWN does not destroy the criticality of the model for intermediate values of the parameter k. In the limits $k \le 2$ and $k \to \infty$ the model on SWN behaves as the same as the 1-d model.

The total payoff of each agent was calculated. The 10 % of agents who have shortcuts are found to have significantly higher payoff than the others. This is because of the presence of the third term in Eq. (3) for calculating the payoff of those agents. This behavior increases the deviation in payoffs which exists in the real economic systems. This contradicts the both cases of the 1-d model (with both Nash and Pareto updating rules), where the payoffs of all agents are close to each other. Also, it implies that the increase of collaborators may increase the payoff, if agents use the Pareto optimality.

3 Application of the Sznajd sociophysics model to SWN

The simple Ising model is one of the fundamental concepts of Statistical Mechanics. Recently, it has been modified to model the problem of two-state opinion formation [12-14]. Depending on an old principle that says "united we stand, divided we fall", the Sznajd model [12] is constructed. Consider a chain of spins $S_i,\ i = 1, 2, 3, \ldots, L$, that are either up $(+1)$ or down (-1). Assume that each pair of adjacent spins can affect the state of their nearest neighbors using the following updating rule:

$$\begin{array}{c} \text{if } S_i S_{i+1} = +1, \quad \text{then } S_{i-1} = S_{i+2} = S_i, \\ \text{if } S_i S_{i+1} = -1, \quad \text{then } S_{i-1} = S_{i+1},\ S_{i+2} = S_i. \end{array} \tag{5}$$

Simulating this system for long time, where at each time step the site i is selected randomly, one obtains finally one of the following three fixed points: $\uparrow\uparrow\uparrow \ldots$, $\downarrow\downarrow\downarrow \ldots$, $\uparrow\downarrow\uparrow\downarrow \ldots$ with probability 0.25, 0.25, 0.5, respectively. Then

the model has two possible final states: dictatorship or stalemate, so no common decision in a democratic way can be done. Only after introducing a small finite noise, a democratic decision can be done.

This model is generalized to 2-d [13] obtaining similar fixed points, except in some variants. Also, it is used to explain the distribution of votes among candidates in the Brazilian local elections [14].

We generalized the Sznajd model to SWN [7]. The updating rule is generalized to include the shortcutting neighbors, if exist, as follows:

$$\begin{array}{c} \text{if } S_i S_{i+1} = +1, \quad \text{then } S_{i-1} = S_{i+2} = S_{sc(i)} \text{ (if exists)} \\ = S_{sc(i+1)} \text{ (if exists) } = S_i, \\ \text{if } S_i S_{i+1} = -1, \quad \text{then } S_{i-1} = S_{sc(i)} \text{ (if exists) } = S_{i+1}, \\ S_{i+2} = S_{sc(i+1)} \text{ (if exists) } = S_i \end{array} \tag{6}$$

where $sc(i)$ is the shortcutting neighbor of the i-th vertex, if exists.

Beginning with a totally random initial state, the system always reaches one of two fixed points all up or all down with equal probabilities. Also, the effects of the initial concentration of the up and down spins is studied [7]. Although the stalemate fixed point disappeared in this model, the model always evolves to limiting dictatorship fixed points. So no democratic decision can be taken in this system. This point is improved by introducing the concept of a leader.

A leader is usually a strong person who completely trusts in his/her opinion. Also, he/she is always capable in influencing somebody to follow his/her thought without changing. As a simple definition for a leader, we define a leader as a person who does not change his/her opinion and has the capability of influencing one of his/her nearest neighbors to accept the same opinion as the leader.

We randomly choose two persons to be leaders, one for the up direction and the other to the down direction. The updating rule is the same as Eq. (6). The same numerical investigation is reapplied to this modified model. No fixed point is observed in this system. The time evolution of the magnetization, ($M = \frac{1}{L}\sum_{i=1}^{L} S_i$) is drawn [7]. Sometimes the majority of the population follow the up direction, and another times the majority follow the down direction. There is no dependence on the initial concentrations for the up and down followers. There is no periodicity observed. Also, this behavior is independent of the lattice size.

Because of the disappearing of the dictatorship fixed points, a decision can be done in a democratic way. This behavior is clearly observed in many local

elections in many democratic countries. For example, sometimes the majority is for the Democratic party and others the majority is for the Republic party, without reaching a state at which 100% of the population follow a certain opinion. Also, this behavior is not periodic. Similar behavior is observed from both 1-d and 2-d versions of Sznajd model after including some noises.

An interesting question is what will happen if one of the two leaders is killed or forced to slough his leadership by any way? In this case the system evolves into a single dictatorship fixed point following the opinion of the leader that is still alive, independent of the concentrations of each opinion at the killing process. This explains why dictators usually kill the active objectors members, and gives a warning to leaders, they have to prepare some leaders for the future. The model is investigated using time series analysis [7].

4 Conclusions

In conclusion, SWN is a good description for real social networks. It allows some simple mathematical models to display some interesting socio-economic aspects.

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References

[1] Everett M. Rogers, Diffusion of Innovations (The Free Press, New York, 1995).

[2] M. E. J. Newman and D. J. Watts, Phys. Lett. A 263, 341 (1999).

[3] D. J. Watts and S. H. Strogatz, Nature 393, 440 (1998).

[4] E. Ahmed and A. S. Elgazzar, Europ. Phys. J. B 18, 159 (2000).

[5] E. Ahmed and A. S. Elgazzar, Int. J. Mod. Phys. C 11, 1539 (2000).

[6] E. Ahmed, A. S. Hegazi and A. S. Elgazzar, Int. J. Mod. Phys. C 13, 189 (2002).

[7] A. S. Elgazzar, Int. J. Mod. Phys. C 12, 1537 (2001).

[8] A. S. Elgazzar, Physica A 303, 154 (2002).

[9] D. Zajdenweber, Scale Invariance in Economy and Finance, in "Scale Inveriance and Beyond", Edited by B. Dubrulle, F. Graner and D. Sornette (EDP Sciences and Springer, Berlin, 1997).

[10] A. Arenas, A. Dias-Guilera, C. J. Perez and F. Vega-Redondo, Phys. Rev. E 61, 3466 (2000).

[11] J. W. Friedman, Oligopoly and the Theory of Games (North-Holland Publishing Co., Amsterdam, 1977).

[12] K. Sznajd-Weron and J. Sznajd, Int. J. Mod. Phys. C 11, 1157 (2000).

[13] D. Stauffer, A. O. Sousa and S. M. de Oliveira, Int. J. Mod. Phys. C 11, 1239 (2000).

[14] A. T. Bernardes, U. M. S. Costa, A. D. Araujo, and D. Stauffer, Int. J. Mod. Phys. C 12, 159 (2001); D. Stauffer, Computer Physics Communications 146, 93 (2002).

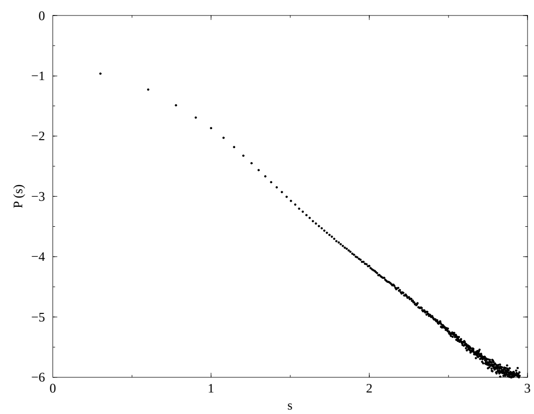

Figure 1: A log-log plot for the probability distribution function of avalanches' sizes $P(s)$ versus the avalanches' sizes s for the 1-d model using Pareto optimality. A power law behaviour is obtained with exponent $\gamma = 1.99$, for $k = 4$. The results are averaged over ten independent runs on a 1-d lattice of size 1000 updated for 6×10^6 time steps.